Exercise 15

Prove the statement using the ε , δ definition of a limit and illustrate with a diagram like Figure 9.

$$\lim_{x \to 3} \left(1 + \frac{1}{3}x \right) = 2$$

Solution

According to Definition 2, proving this limit is logically equivalent to proving that

if
$$|x-3| < \delta$$
 then $\left| \left(1 + \frac{1}{3}x \right) - 2 \right| < \varepsilon$

for all positive ε . Start by working backwards, looking for a number δ that's greater than |x-3|.

$$\left| \left(1 + \frac{1}{3}x \right) - 2 \right| < \varepsilon$$
$$\left| \frac{1}{3}x - 1 \right| < \varepsilon$$
$$\left| \frac{1}{3}(x - 3) \right| < \varepsilon$$
$$\frac{1}{3}|x - 3| < \varepsilon$$
$$|x - 3| < 3\varepsilon$$

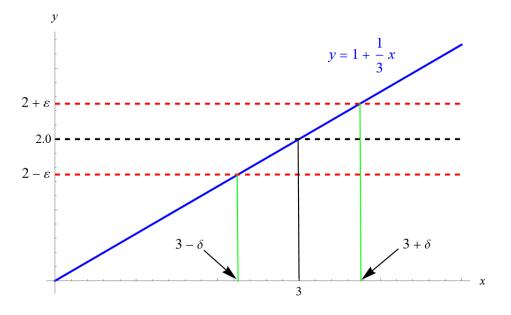
Choose $\delta = 3\varepsilon$. Now, assuming that $|x - 3| < \delta$,

$$\left| \left(1 + \frac{1}{3}x \right) - 2 \right| = \left| \frac{1}{3}x - 1 \right|$$
$$= \left| \frac{1}{3}(x - 3) \right|$$
$$= \frac{1}{3}|x - 3|$$
$$< \frac{1}{3}\delta$$
$$= \frac{1}{3}(3\varepsilon)$$
$$= \varepsilon.$$

Therefore, by the precise definition of a limit,

$$\lim_{x \to 3} \left(1 + \frac{1}{3}x \right) = 2.$$

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Below is an illustration like Figure 9.